DEVELOPMENT OF TAYLOR–GOERTLER VORTICES IN THE BOUNDARY LAYER OF A SURFACE MOVING CURVILINEARLY IN THE PRESENCE OF POLYMER ADDITIVES

O. D. Nikishova and N. A. Gorbatyuk

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We present results from experimental studies into the effect of weak polymer solutions on flow stability in a boundary layer with curvilinear streamlines.

We know that additions of small quantities of long-chain linear polymers leads to a reduction in hydrodynamic resistance in the case of turbulent flow. The great practical significance of this effect led to numerous studies, reviews of which can be found in [1-3]. The complexity of the processes taking place within a turbulent flow has prevented us, up to the very present, from developing a noncontradictory theory with respect to the Thoms effect. The determinate nature of the processes of loss instability offers some hope that a study of such processes will aid us in ascertaining the physical properties of weak polymer solutions which are responsible for this effect.

In the estimation of the authors of [4-6], the presence of polymer additions retards the laminar flow regime in the boundary layer and leads to a change in the critical Reynolds number. Calculations are presented in [7] and these show that minor additions of a polymer change the critical Taylor number. In the opinion of the authors of [8], the experimental data relating to the increase or reduction in the critical Taylor number can be explained by incorrect viscosity measurements, i.e., at the present time there exists no single opinion with regard to the effect of small polymer additions as affects the stability of shearing flow.

It is the goal of the present study to investigate the influence exerted by weak polymer solutions on the stability of a boundary layer with curvilinear streamlines. Because the forces of pressure and the centrifugal inertial forces in the laminar boundary layer on the distorted surface are not in equilibrium, the flow becomes unstable with respect to the three-dimensional perturbations increasing downstream, said perturbations represented by vortices having axes directed along the velocity of the main flow. When a viscous fluid streamline a concave wall, the flow stability diagram, such as that obtained by Goertler [9], yields $G_{cr} = 0.58$. It has been demonstrated theoretically [10] that in a boundary layer moving about the circumference of a distorted surface in a nonmoving viscous fluid a Taylor–Goertler type instability also appears on the convex side of the trajectory, and in this case we have $G_{cr} = 2.1$. We have also taken note of the difference in the positions of the curves on the stability diagram.

For our experimental studies into the features of the transitional processes occurring in the boundary layer of a distorted surface, we used a rotation device which consisted of a circular basin filled with the fluid, a tow sled rigidly attached to a shaft, positioned above the level of the fluid in the basin. For purposes of visualization and motion-picture photography, the basin was fabricated out of a plastic 15 mm in thickness, in the form of a cylinder with an inside radius of 0.35 m. The tow sled is fashioned out of a crosspiece fitted onto the vertical drive shaft. The crosspiece is set into motion with smooth control over the required velocity range 0.05-0.35 m/sec by means of a direct-current motor with a combined excitation winding. At a specified distance from the center of rotation, the test model was attached to the crosspiece by means of a well-streamlined arm. To prevent the model from entering its own wake, the length of the tow line was limited to a distance of $(3/2)\pi r$.

Measurements showed that the velocity of the plate, following the start, reaches constant values over an angular path equal to 8-12° (depending on the magnitude of the velocity). However, it is obvious that the characteristic magnitudes of the boundary layer in this case do not reach their steady values. In order to estimate the angular length of the path, over which the flow in the boundary layer might be regarded as steady, we will use the solution presented in [11, 12] for the case of sudden plate start. Since the steady boundary-layer regime is gradually extended from the leading to the trailing edge, let us examine what lapse of time is required, for example, for the thickness $\delta = 4\sqrt{\nu t}$ of the nonsteady boundary layer to reach $\delta = 5\sqrt{\nu l/u_0}$. This sought path length is independent of velocity and amounts to $1.5l$. For specific plate lengths this amounts to a quantity smaller than $\pi r/2$. Thus, if we also take into consideration the length of the linear segment, we find that toward the end of the first third of this path the boundary layer attains a steady state with a satisfactory degree of accuracy. Flow visualization in the boundary layer is accomplished

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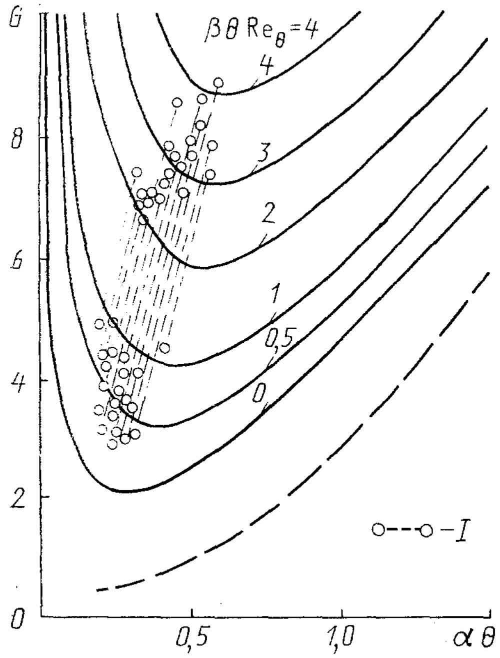

Fig. 1. Flow-stability diagram for the boundary layer of a bent plane moving in a curvilinear direction: I) visually observed instants of the appearance, existence, and destruction of vortex structures; dashed line) curve of neutral stability for the streamlining of the concave surface [9].

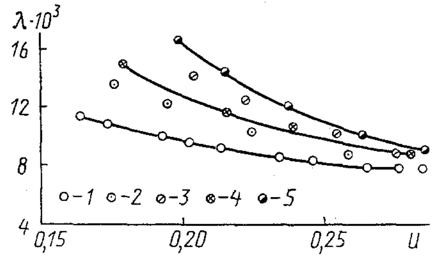

Fig. 2. Average wavelength of the Taylor–Goertler vortices as a function of the velocity of plate motion through water (1) and through aqueous solutions of polyacrylamide with a concentration of $1 \cdot 10^{-5}$ (2), $1 \cdot 10^{-4}$ (3) and in aqueous solutions of biopolymer with concentrations of $1 \cdot 10^{-5}$ (4) and $1 \cdot 10^{-4}$ (5). The plate curvature radius and the radius of curvature for the trajectory of plate motion 0.22 m. λ, m; u, m/sec.

by means of aluminum powder. To illuminate the region of the boundary layer near the convex surface of the model, we created a knife beam curvilinear in shape. Distorted plates with a curvature radius 0.15-0.25 m, evolving into a rectangle, served as the model.

The flow pattern was recorded on film at the end of the tow path. In processing the experimental photographic results, we determined the wavelength λ of the developing perturbations and calculated the wave number $\alpha = 2\pi/\lambda$. From the measured angular frequency of rotation we calculated the linear velocity of plate motion. From the photographs we determined the distance x from the leading edge of the plate to the test segment. We should note that the expression for the calculation of θ is the same as for the plane case [11], whch is valid in satisfying conditions $\delta/r << 1$. This can be demonstrated on the basis of an analysis of the Navier–Stokes equation written in a curvilinear orthogonal coordinate system linked to the trajectory of the moving surface, from which we can find that all of the terms in the equation taking into consideration curvature are smaller by a factor of δ/r than the terms retained in the Prandtl equation. Then, on satisfaction of the additional condition $r >> \delta$ (which was the case in these experiments) they can also be omitted. Consequently, for boundary-layer flow we derive a system consisting of the Prandtl equation for the plane case, plus the equation for the pressure gradient across the boundary layer, containing particles of fluid along the curvilinear trajectory. Therefore, for the characteristics of unperturbed flow in a laminar boundary layer we can make use of the familiar Blasius solution [11] without introducing any significant error into the calculations.

The experimental results have been plotted onto the stability diagram (Fig. 1) taken from [10]. The points denote the instants of visual observation of the onset of vortex structures and their destruction or convergence at the trailing edge of the model. As we can see from Fig. 1, the experimental data yield not only qualitative confirmation of theoretical conclusions with regard to flow instability in the laminar boundary layer of a convex plate moving along a circular trajectory, but they also yield satisfactory quantitative agreement with theoretical and experimental results. Linear theory predicts the presence of Taylor—Goertler vortices in a boundary layer at $G > 2.1$. Under the conditions of the experiment, the Taylor—Goertler vortices appear clearly and were recorded on film at $G > 2.8$.

Experimental investigation of the effect that is exerted by polymer additions on the vortex instability in the boundary layer were conducted with volumetric additive concentrations ranging from $1 \cdot 10^{-6}$ to $1 \cdot 10^{-4}$. To eliminate the effect of additives unaccounted for in the solution, each experiment was begun with control photographs of the flow in the boundary layer of the plate in water that had been poured into a carefully cleansed basin. The hydrodynamic effectiveness of the polymer was tested by means of an express-analysis method involving the use of a capillary unit [13], with the polymer then added to the basin, starting with a polymer concentration of $1 \cdot 10^{-6}$ in the solution. At the conclusion of the experiments, at a given concentration, we increased the polymer concentration to $5 \cdot 10^{-6}$ in the basin water, then to $1 \cdot 10^{-5}$, etc., all the way to $1 \cdot 10^{-4}$.

After we had processed the photographs, we determined that the presence of such additives significantly retards the transitional process in the boundary layer of the plate. In this range of velocities of plate motion in the basin and depending on the polymer concentration in the solution, the location of vortex onset and their wavelengths change. From the results of the film processing we constructed graphs showing the vortex wavelength as a function of the linear velocity of the model and the concentration of the polymer in the solution. Figure 2 shows these data for a model with a curvature radius 0.22 m, moving through water, through solutions of polyacrylamide, and through a biopolymer [13]. We can see from Fig. 2 that the different polymers under identical conditions exhibit qualitatively identical effects with respect to the formation of vortex instability of the Taylor—Goertler wave type in the boundary layer of the plate. Some of the quantitative differences can be explained if we take into consideration the form of the polymer, its molecular mass, and a number of other characteristic properties of these additives.

In order to ascertain the effect exerted by a change in the viscosity of the solution on the phenomena being studied here, in parallel with the experiments we conducted control measurements of the viscosity of the solutions with various polymer concentrations. The measurements showed that the magnitude of the relative biopolymer solution viscosity amounted to 1.001 for a concentration of $1.0 \cdot 10^{-6}$ to 1.003 for a concentration of $1.5 \cdot 10^{-4}$. No such slight change in viscosity can significantly alter the characteristic parameters of the boundary layer, not to speak of θ, since the viscosity in the expression for θ is in a ratio of 1/4. Consequently, the change in the viscosity of the polymer solution cannot be responsible for the observed effect.

As follows from the derived experimental results, the wavelength of the Taylor—Goertler vortices changes significantly in polymer solutions, although at the concentrations examined here the parameters of the boundary layer are virtually identical. This suggests that there is a change in the critical Goertler number. In this event, if we assume that the ratio of the wavelength for the fastest growing perturbation to the thickness of the boundary layer in pure water and in a polymer solution is one and the same ($\alpha\theta$ = 0.2-0.3 from Fig. 1) at the point at which the Goertler number reaches its critical value, then the critical Goertler number in the polymer solution can apparently be estimated from the relationship

$$G_{crw}/G_{crw} \sim (\lambda_w/\lambda_p)^{3/2} .$$

The derived experimental results confirm the hypotheses put forward in [14], to the effect that polymer additions must increase the critical Goertler number and the wavelength of the perturbation. This conclusion on the part of the authors of [14] was based on solution of the problem in a linear formulation with respect to flow stability along the concave surface of a fluid to which the hypothetical polymer has been added, thereby elevating the value of the longitudinal viscosity.

It should be noted that at low velocities for plate motion the effect of the polymer additions on the wavelength is more clearly defined than in the case of high velocities. This may be attributed to the effect of roughness. Figure 3 shows photographs of the flow in the boundary layer of a plate exhibiting local roughness in its central portion 15 mm from the leading edge. The photograph yields information with respect to the formation of the Taylor—Goertler vortices in the boundary layer of the plate as it moves at a velocity 0.23 m/sec through water (Fig. 3a) and in a polymer solution (Fig. 3b). We can see from Fig. 3a that in the peripheral zones of the plate surface, where no local roughness is to be found, the Taylor—Goertler vortices, arising at some distance from the leading edge of the plate, retain their structure virtually to the instant at which they move away at the trailing edge of the plate.

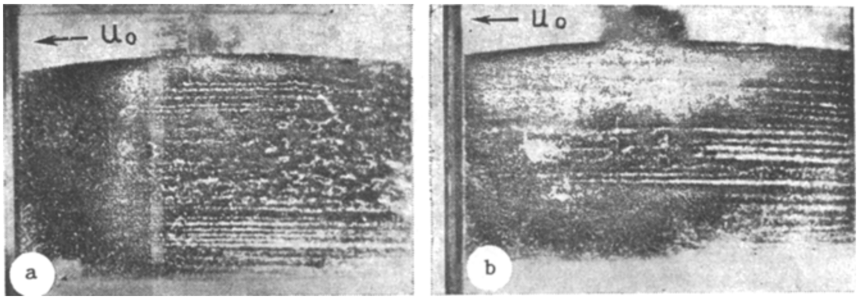

Fig. 3. Formation of Taylor—Goertler vortices in the boundary layer of a bent plate moving through a fluid along the arc of a circle with a radius of 0.22 m: a) through water; b) in an aqueous solution of polyacrylamide with a concentraton of $C = 5 \cdot 10^{-5}$; the linear velocity of the model is 0.23 m/sec; the curvature radius of the model is 0.22 m.

In the central zone of the plate, owing to the effect of local roughness, the vortices begin intensively to disintegrate, almost immediately after formation.

The addition of the polyacrylamide significantly altered the pattern of formation and vortex development. This is particularly noticeable in the central zone of the plate (Fig. 3b) where the Taylor—Goertler vortices, in contrast to the case of a surface moving through water, exhibit a clearly defined structure and converge with the trailing edge without disintegration. In the peripheral regions of the model, the influence of polymer additions can be detected in the fact that the vortices arise later and have a somewhat greater wavelength in comparison to the case of motion through water.

Thus, based on the results of the research carried out, we can draw the following conclusions:

1. In the movement of a distorted surface along a circular trajectory, the flow in the boundary layer is unstable relative to the vortex perturbations; the region of instability is described by the diagram derived in [10].

2. The addition of high-molecular polymers leads to an increase in the critical Goertler number and in the wavelength of the perturbations, as well as to a reduction in the destabilizing effect of surface roughness.

NOTATION

$G = (u_0\theta/\nu)\sqrt{\theta/r}$, Goertler number; u_0, linear velocity of the model; θ, momentum loss thickness for the boundary layer; r, curvature radius of model surface; l, length of model evolution; ν, kinematic coefficient of viscosity; x, distance from the leading edge of the model; α, wave number; λ, wavelength of the Taylor—Goertler vortices; $G_{cr} = (u_0\theta/\nu)\sqrt{\theta/r}$, critical Goertler number; β, parameter characterizing perturbation growth; Re_θ, Reynolds number, calculated from the formula $Re_\theta = u_0\theta/\nu$; δ, boundary layer thickness. Subscripts: w, water; p, aqueous polymer solution; cr, critical number.

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